QP CODE: 26007653

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Reg No : .....

Name : .....

B.Sc DEGREE (CBCS)SPECIAL REAPPEARANCE EXAMINATIONS, JANUARY 2026

Fifth Semester

CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

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Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that set of all even numbers is denumerable?
2. Prove that there doesnot exist any smallest positive real number?
3. Prove that
   (a.) $|a|^2 = a^2 \ \forall a \in R$
   (b.) $|ab| = |a||b|, \ \forall a, b \in R$
4. Define Nested intervals? Is the interval $I_n = \left(-\frac{1}{n}, \frac{1}{n}\right), n \in N$ nested?
5. Show that $lim(\frac{1}{3^n}) = 0$.
6. Find $lim(\frac{(-1)^n}{n+2})$.
7. Using Monotone Convergence Theorem, prove that $lim(\frac{1}{\sqrt{n}}) = 0$.
8. Prove that $((-1)^n)$ is not Cauchy.
9. If c > 1, prove that lim $(c^n) = +\infty$.
10. Show that the series $\sum \frac{\cos n}{n^2}$ is convergent.
11. Define an Alternating Series. Give an example.
12. Show that $\lim_{x \to c} x = c$ for any $c \in \mathscr{R}$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that If A, B are bounded sets then $Sup\ (A+B) = Sup\ A + Sup\ B$ where $A+B = \{a+b : a \in A, b \in B\}$

14. Prove that $x \in [0,1]$ then the binary representation of x forms a sequnce consisting only $0, 1$?

15. Using definition of limits, prove that $lim(\frac{3n+2}{n+1}) = 3.$

16. Let X = (x_n) and Y = (y_n) be sequences of real numbers that converges to x and y respectively. Prove that the sequences X+Y converges to x+y.

17. Let X = (x_n) be a sequence of real numbers. Prove that the following statements are equivalent.
    (i) The sequence X = (x_n) does not converge.
    (ii) There exists an ϵ_0>0 such that for any natural number k, there exists an n_k such that $n_k \geq k$ and $|x_{nk} - x| \geq \epsilon_0$.
    (iii) There exists an ϵ_0>0 and a subsequence X' = (x_{nk}) of X such that $|x_{nk} - x| \geq \epsilon_0$ for every natural number k.

18. State and prove root test.

19. If Σa_n is a convergent series of real numbers then is it necessary that $\Sigma \frac{\sqrt{a_n}}{n}$ is convergent?

20. If $A \subseteq \mathscr{R}$ and $f : A \to \mathscr{R}$ has a limit at $c \in \mathscr{R}$, then prove that f is bounded on some neighborhood of c.

21. Give an example of a function that has a left-hand limit but not a right-hand limit at a point.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that there exist a real number x such that $x^2 = 2$?

23. (a) State Monotone Convergence Theorem.
    (b) Prove that for any real number a > 0, there exists a sequence (s_n) of real numbers that converges to $\sqrt{a}$.
    (c) Use the above sequence to evaluate the value of $\sqrt{5}$ correct to 5 decimal places.

24. (a) State and prove Rearrangement Theorem for the series in R.

(b) Comment on "A series in R is convergent if and only if it is absolute convergent"

25. (a) Let $A \subseteq \mathscr{R}$, $f, g : A \to \mathscr{R}$, and let $c \in \mathscr{R}$ be a cluster point of A, Suppose that $f(x) \leq g(x)$ for all $x \in A$, $x \neq c$, Then prove the following

- If $\lim_{x \to c} f = \infty$, then $\lim_{x \to c} g = \infty$.
- If $\lim_{x \to c} g = -\infty$, then $\lim_{x \to c} f = -\infty$.

(b) Give an example of a function that has a left-hand limit but not a right-hand limit at a point.

(c) Evaluate the limit or show that it do not exist " $\lim_{x \to 1} \frac{x}{x-1}$ where $x \neq 1$.

(2×15=30)